NUMERICAL SIMULATION OF RECIRCULATING FLOW NEAR A GROYNE

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ABSTRACT: A two-dimensional depth-averaged shallow-water-equation model is presented in the paper. A depth-averaged $k$-$\varepsilon$ model is used as the turbulence closure model. The model is used to simulate the recirculating flow near a groyne. In order to improve the prediction for the recirculating length, the adjustment of empirical coefficient $C_\mu$ is adopted in the turbulence model. The computed recirculating length agrees well with the experimental measurement. In addition, the numerical results of velocities and bed shear stresses are compared with the available experimental data. A reasonable agreement has been obtained.

1. INTRODUCTION

In hydraulic and coastal engineering, groynes are important structures for river navigation, shoreline protection and beach reclamation. Many researches have been conducted experimentally and numerically to investigate the flow pattern and scouring in the vicinity of the groynes\(^ [1][2]\). Due to the variety of groynes, i.e., relative length to the water space, inclination angle to the flow direction, submerged or non-submerged form, groyne numbers, et al., flow separation and recirculating length would be greatly different, which is a challenge to the applications of numerical models.

Since the standard $k$-$\varepsilon$ turbulence model often gives an underprediction for the recirculating length when the streamlines are strongly curved, some special treatments have been employed by researchers in the computation of flow field. Tingsanchali and Maheswaran\(^ [3]\), for example, modified the convective and diffusive flow fluxes at the meshes near groyne to get more realistic results. Furthermore, they used the streamline curvature correction to modify the dissipation term in $\varepsilon$-equation. As a result, the prediction for the recirculating length was improved. A simpler method used by Chapman and Kuo\(^ [4]\), however, adopted a decreased empirical coefficient $C_\mu$ to solve this problem.

In this paper, a shallow-water-equation model\(^ [5]\) has been extended firstly to include the depth-averaged $k$-$\varepsilon$ turbulence closure model for turbulent flow modeling. Then this model is used to calculate the flow field near a groyne in the open channel. In order to improve the prediction of recirculating length, adjustment of empirical coefficient $C_\mu$ is used in the turbulence model. The numerical results are compared with the experimental data and the analysis for the results is given.

2. FORMULATION OF THE MODEL

The depth-averaged 2D shallow-water equations have the following forms in a Cartesian coordinate system:

\[
\begin{align*}
\frac{\partial H}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} &= 0 \\
\frac{\partial P}{\partial t} + \frac{\partial}{\partial x} \left( \frac{P^2}{H} \right) + \frac{\partial}{\partial y} \left( \frac{PQ}{H} \right) + gH \frac{\partial \eta}{\partial x} &= \frac{1}{\rho} \tau_{nx} + \frac{1}{\rho} \frac{\partial (HT_{sx})}{\partial x} + \frac{1}{\rho} \frac{\partial (HT_{sy})}{\partial y} \\
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{PQ}{H} \right) + \frac{\partial}{\partial y} \left( \frac{Q^2}{H} \right) + gH \frac{\partial \eta}{\partial y} &= \frac{1}{\rho} \tau_{ny} + \frac{1}{\rho} \frac{\partial (HT_{sx})}{\partial x} + \frac{1}{\rho} \frac{\partial (HT_{sy})}{\partial y}
\end{align*}
\]

where $H(=\eta + z_s)$ is the total water depth; $\eta$ is the free surface elevation with respect to the datum; $z_s$ is
the bed elevation with negative value; $U$ and $V$ are the depth-averaged velocity components in $x$- and $y$-directions, respectively; $P(=UH)$ and $Q(=VH)$ denote the $x$- and $y$- components of the volume flux, respectively; $g$ is the gravitational acceleration; $\rho$ is fluid density.

The bottom friction stresses, $\tau_{bx}$ and $\tau_{by}$, are written in terms of Manning’s formula

$$
\tau_{bx} = \frac{\rho g n^2}{H^{\frac{2}{3}}} P \sqrt{P^2 + Q^2}
$$

$$
\tau_{by} = \frac{\rho g n^2}{H^{\frac{2}{3}}} Q \sqrt{P^2 + Q^2}
$$

in which $n$ is the Manning’s roughness coefficient. $T_{xx}$, $T_{yy}$, and $T_{xy}$ are the depth-averaged Reynolds stresses which can be closed based on the Boussinesq eddy viscosity hypothesis

$$
\frac{1}{\rho} HT_y = v_i \left[ \frac{\partial (HU_i)}{\partial x_j} + \frac{\partial (HU_j)}{\partial x_i} \right] - \frac{2}{3} k H \delta_{ij} \delta_y
$$

where $i, j = 1, 2$ representing $x$- and $y$-directions, respectively, $\delta_y$ is the Kronecker delta ($\delta_y = 1$ for $i = j$ and $\delta_y = 0$ for $i \neq j$) and $v_i$ is the depth-averaged turbulent viscosity which is the function of depth-averaged turbulent kinetic energy $k$ and its dissipation rate $\varepsilon$

$$
v_i = C_{\mu} \frac{k^2}{\varepsilon}
$$

in which $C_{\mu}$ is an empirical constant.

The distribution of $k$ and $\varepsilon$ over the flow field can be determined from the following semi-empirical transport equations for $k$ and $\varepsilon$:

$$
\frac{\partial (Hk)}{\partial t} + \frac{\partial (HUk)}{\partial x} + \frac{\partial (HVk)}{\partial y} = \frac{\partial}{\partial x} \left[ v_i \frac{\partial (Hk)}{\partial x} \right]
$$

$$
+ \frac{\partial}{\partial y} \left[ v_i \frac{\partial (Hk)}{\partial y} \right] + P_t + P_{rv} - \varepsilon H
$$

where $\sigma_x, \sigma_y, C_{\mu} \varepsilon$ and $C_{2\varepsilon}$ are further empirical constants, the production terms have the following expressions

$$
P_t = v_i \left[ \frac{\partial (HU)}{\partial x} \right]^2 + 2 \left[ \frac{\partial (HV)}{\partial y} \right]^2
$$

$$
+ \left[ \frac{\partial (HU)}{\partial y} + \frac{\partial (HV)}{\partial x} \right]^2
$$

$$
P_{rv} = \frac{1}{\sqrt{C_f}} u_i^* \text{ and } P_{rv} = 3.6 \frac{C_{2\varepsilon}}{C_{\varepsilon}} \frac{C_{2\varepsilon}}{C_{\varepsilon}} \frac{u_i^*}{H}
$$

where $u_i = \sqrt{C_f (U^2 + V^2)}$ with friction coefficient $C_f = n g / H^{0.5}$ for rough beds.

The empirical constants mentioned above can take the following values recommended by Launder and Spalding:

$$
C_{\mu} = 0.09, \sigma_x = 1.0, \sigma_y = 1.3,
$$

$$
C_{2\varepsilon} = 1.44, C_{2\varepsilon} = 1.92
$$

3. INITIAL AND BOUNDARY CONDITIONS

At the initial moment, the flow motion can be assumed to be zero, i.e., $P=Q=0$. For the turbulence field, small finite values are necessary as the “seed” of the initial disturbance. According to Lin and Liu, $k = u_i^2 / 2$ with $u_i = \delta U_0$ where $U_0$ is the inflow velocity and $\delta$ is chosen as $2.5 \times 10^{-3} \varepsilon$. $\varepsilon$ is estimated through equation $\varepsilon = C_{\varepsilon} k^2 / v_i$ with $v_i = \xi \nu$, where $\xi$ is chosen to be 5.0 at present.

The flow rate must be specified at the upstream boundary, i.e., $P=P_0$, $Q=0$, whereas depending on the flow state, the water depth boundary condition, i.e., $H=H_0$, would be specified at the downstream boundary for subcritical flow and at the upstream for supercritical
flow. In the region near walls, no-slip boundary condition should be applied by setting the velocity at the solid boundary equal to zero, i.e., \( P_s = P_e = 0 \) \[8\]. Besides, the conditions are specified at a grid point inside the turbulent boundary layer by logarithmic law-of-the-wall

\[
\frac{U_g}{u_*} = \frac{1}{k} \ln \left( \frac{H}{y_0} \right)
\]

(12)

where \( U_g \) is the resultant velocity parallel to the wall, \( k = 0.41 \) is the von Karman constant, \( E \) is a parameter accounting for the wall roughness (=9.0 for hydraulically smooth walls), \( y_0 \) is the normal distance to the walls. This law should be applied to a point in the range \( 0.41 \leq \frac{y_0}{u_*} \leq 30 \) \[9\]. Under the equilibrium assumption of turbulence near walls, the solid boundary conditions for \( k \) and \( \varepsilon \) are given by \[3\]

\[
k = \frac{u_*^2}{\sqrt{C_p}} \quad \text{and} \quad \varepsilon = \frac{u_*^4}{\kappa^2 y_0^2}
\]

(13)

4. NUMERICAL IMPLEMENTATION

In this study, the finite difference method constructed in the staggered grid system will be used throughout the computation with all scalar quantities, i.e., \( \eta, H, k \) and \( \varepsilon \), defined at the center of the cells and the vectors, i.e., \( U, V, P \) and \( Q \), defined at the interfaces of the cells.

4.1 SHALLOW-WATER EQUATIONS

The shallow-water equations without nonlinear terms are discretized with the explicit leap-frog finite difference scheme, whereas the nonlinear convection terms with an upwind scheme. Since the upwind scheme is employed, the discretized momentum equations are only first order in accuracy in terms of spatial grid sizes. The stress terms are discretized with central difference scheme with \( T_{xx} \) and \( T_{yy} \) evaluated at the center of the cell and \( T_{xy} \) and \( T_{yx} \) evaluated at the top right corner grid. In addition, the bottom frictional terms are discretized as \[10\]

\[
\frac{1}{\rho} T_{ux} = v_x \left( P_{xx}^{n+1/2} + P_{xx}^{n-1/2} \right)
\]

\[
\frac{1}{\rho} T_{uy} = v_y \left( Q_{xy}^{n+1/2} + Q_{xy}^{n-1/2} \right)
\]

(14)

in which \( v_x \) and \( v_y \) are given in terms of Manning's formula

\[
v_x = \frac{1}{2} \frac{g}{g} \left[ \left( P_{xx}^{n+1/2} \right)^2 + \left( Q_{xy}^{n+1/2} \right)^2 \right]^{3/2}
\]

\[
v_y = \frac{1}{2} \frac{g}{g} \left[ \left( P_{xx}^{n+1/2} \right)^2 + \left( Q_{xy}^{n+1/2} \right)^2 \right]^{3/2}
\]

(15)

where subscript \((i, j)\) denotes spatial nodes, superscript \(n\) denotes time level. More details about the finite difference forms of shallow-water equations can be found from [5].

4.2 DEPTH-AVERAGED \( k - \varepsilon \) MODEL

Both \( k \)- and \( \varepsilon \)-equations are treated in a semi-implicit way and can be symbolically written as \[8\]

\[
\frac{(H \varepsilon)^{n+1/2} - (H \varepsilon)^{n-1/2}}{\Delta t} = \frac{F_k X + F_k Y}{\Delta t}
\]

\[
= VIS \varepsilon X + VIS \varepsilon Y + C_1 \frac{(\varepsilon)^{n+1/2}}{k^{n+1/2}} P_{hi, j}^{n+1/2} + C_2 \frac{(\varepsilon)^{n+1/2}}{k^{n+1/2}} H_{hi, j}^{n+1/2} - P_{xy} - C \frac{(\varepsilon)^{n+1/2}}{k^{n+1/2}} H_{xy}^{n+1/2}
\]

\[
\frac{(H k)^{n+1/2} - (H k)^{n-1/2}}{\Delta t} = F_k X + F_k Y
\]

\[
= VIS k X + VIS k Y + P_{hi, j}^{n+1/2} + P_{kxy} - C_1 \frac{(k)^{n+1/2}}{v_{hi, j}^{n+1/2}} H_{hi, j}^{n-1/2}
\]

(16)

Therefore, the final finite difference forms for \( k - \varepsilon \) equations are written as follows

\[
\frac{(H \varepsilon)^{n+1/2} - (H \varepsilon)^{n-1/2}}{\Delta t} = \frac{F_k X + F_k Y}{\Delta t}
\]

\[
= VIS \varepsilon X + VIS \varepsilon Y + C_1 \frac{(\varepsilon)^{n+1/2}}{k^{n+1/2}} P_{hi, j}^{n+1/2} + C_2 \frac{(\varepsilon)^{n+1/2}}{k^{n+1/2}} H_{hi, j}^{n+1/2} - P_{xy} - C \frac{(\varepsilon)^{n+1/2}}{k^{n+1/2}} H_{xy}^{n+1/2}
\]

\[
\frac{(H k)^{n+1/2} - (H k)^{n-1/2}}{\Delta t} = F_k X + F_k Y
\]

\[
= VIS k X + VIS k Y + P_{hi, j}^{n+1/2} + P_{kxy} - C_1 \frac{(k)^{n+1/2}}{v_{hi, j}^{n+1/2}} H_{hi, j}^{n-1/2}
\]

Session D
\[ \epsilon_{ij}^{n+1/2} = \left( \frac{(H \varepsilon)_{ij}^{n+1/2}}{\Delta t} - F_{ij}X - F_{ij}Y + VIS_{ij}X + VIS_{ij}Y + C_{ij} \frac{\epsilon_{ij}^{n+1/2}}{\kappa_{ij}^{n+1/2}} P_{ij}^{n+1/2} + P_{ij} \right) \]  
\[ + VIS_{ij} + \frac{H_{ij}^{n+1/2}}{\Delta t} + C_{2e} \frac{\epsilon_{ij}^{n+1/2}}{\kappa_{ij}^{n+1/2}} H_{ij}^{n+1/2} \]  

5. MODEL TESTS

5.1 EXPERIMENTAL SETUP

Rajaratnam and Nwachukwu [2] conducted the experiments to study the characteristics of the time-averaged turbulent flow near thin plate groyes projecting perpendicularly into a fully developed turbulent flow in a long rectangular channel. The flume used in the experiments was 37m long, 0.9m wide and 0.76m deep with smooth bed and sides. The groyne was an aluminum plate with 3mm thickness and 152mm length and projected well above the water surface. The flow velocity and water depth in Test A1 were \( U_0 = 0.253 \text{m/s} \) and \( H = 0.189 \text{m} \), respectively. The recirculating length measured in the experiment is about 12.5b, where \( b = 152 \text{mm} \) is the length of the groyne. As shown in figure 3 and figure 4, the resultant velocity and bed shear stress profiles measured at \( y/b = 1.0, 1.5, 2.0, 3.0 \) and 4.0 are plotted after normalized by \( U_0 = 0.253 \text{m/s} \) and \( \tau_0 = 0.1293 \text{N/m}^2 \) measured in upstream region, respectively.

5.2 NUMERICAL SETUP AND RESULTS

The computational domain is 6m in length and 0.9m in width. The upstream and downstream boundaries are located at 2m and 4m away from the groyne, respectively. In order to catch the high gradient near the groyne, the non-uniform grid (200 by 60) is packed near the groyne with the minimum grid size 1.5mm in both x- and y-directions (see figure 1). Flow flux of 0.047817m\(^2\)/s and water depth of 0.189m are specified at the upstream and downstream boundaries, respectively. At the channel walls, no-slip boundary condition is applied. Manning coefficient \( n \) is assigned as a typical value for smooth bottom, \( 0.01 \text{s/m}^{1/3} \), in the present computation. In addition, the time step \( \Delta t = 0.0003 \text{s} \).

The recirculating length obtained from the model is 9.43b (see figure 2(a)), which is much smaller than the experimental measurement. The underprediction may attribute to the overprediction of the depth-averaged eddy viscosity in the region of strong streamline curvature. Some researchers adopted the streamline curvature modification to adjust the empirical coefficient \( C_\mu \). Chapman and Kuo [4] suggested that \( C_\mu \) should be significantly reduced in the region of strong streamline curvature. By decreasing \( C_\mu \), they obtained good prediction for the recirculating length of flow past a rearward-facing step.

In this study, we attempt to improve computed recirculating length by adjusting the value of \( C_\mu \). After trial and error, \( C_\mu \) is selected as 0.04. The calculated recirculating length has been improved greatly and reaches 12.7b (see figure 2(b)) which agrees well with the experimental measurement.
The calculated resultant velocity profiles are plotted in figure 3. The comparison between the numerical results and experimental data shows reasonable agreement. A large discrepancy, however, occurs at \( y/b = 2.0 \), where the calculated results underpredict the experimental data in the downstream region of the groyne. It is noted that the numerical results reported by Molls et al.\(^1\) and Tingsanchali and Maheswaran\(^3\) also underpredicted largely in this region. This may be due to the very high velocity gradient arising in this region which makes the depth-averaged model inapplicable. Another possible reason may come from the experimental measurement errors.

The comparison between the calculated bed shear stress and experimental measurement is also given in the study (see figure 4). It can be seen that the bed stress reaches the maximum value at the tip of groyne. However, the model underpredicts this maximum stress and also the stresses in the region downstream the groyne at \( y/b = 1.0, 1.5 \) and \( 2.0 \). The discrepancy here may attribute to the assumption of steady and uniform flow for Manning equation.

On the other hand, comparing the numerical results when \( C_\mu \) has different value, we can find that the adjustment of \( C_\mu \) will not affect the velocity fields in the region upstream the groyne. Even in the region of \( x/b < 2.0 \) downstream the groyne, the influence from \( C_\mu \) is negligible. However, the velocity field begins to differ in the further downstream region where the velocity gradient is much higher. The determination of \( C_\mu \) and even the applicability of the depth-averaged \( k-\varepsilon \) turbulence model in the situation of high velocity gradient need to be examined carefully in the next step of study.

6. CONCLUSIONS

In the paper a two-dimensional depth-averaged shallow-water-equation model is presented. A depth-averaged \( k-\varepsilon \) model is used as the turbulence closure. When this numerical model is used to simulate the recirculating flow near a groyne, the recirculating length has been underpredicted. This may attribute to the overprediction of the depth-averaged eddy viscosity in the region of recirculation. In order to improve the prediction for the recirculating length, the adjustment of empirical coefficient \( C_\mu \) is adopted in the turbulence model. The computed velocity profiles are compared with the available experimental data. A reasonable agreement can be obtained. Further works are needed to examine the determination of \( C_\mu \) and the applicability of the depth-averaged \( k-\varepsilon \) turbulence model in the situation of high velocity gradient.

REFERENCES

Figure 1: Grid arrangement in the computational domain; groyne is located at $x=2\text{m}$; lines are plotted every two grid nodes for easier visibility.

Figure 2: Computed streamline pattern: (a) when $C_\mu=0.09$; (b) when $C_\mu=0.04$; $x/b=0$ is the groyne position along flume direction.
Figure 3: Comparison between computed resultant velocity profiles and experimental data \[^{[2]}\]; all the velocities are normalized by $U_0=0.253\text{m/s}$; $x/b=0$ is the groyne position along flume direction.

Figure 4: Comparison between computed bed shear stress profiles and experimental data \[^{[2]}\]; all the shear stresses are normalized by measured $\tau_0=0.1293\text{N/m}^2$ in upstream region; $x/b=0$ is the groyne position along flume direction.