QUALITATIVE AND QUANTITATIVE FEATURES OF WAVE BREAKING OVER A SUBMERGED BREAKWATER, AND EFFECTS ON NONLINEAR WAVE-STRUCTURE INTERACTION

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SUMMARY
Experimental tests on interaction between a wave and a submerged rubble-mound breakwater were performed in wave flume at University of Napoli Federico II. Different incident wave conditions were tested, including both non-breaking and breaking waves over the barrier. In cases of breaking, different breaker types at structure were described and classified according to hydraulic-morphological criteria. Experimental results show, according to early literature, that wave breaking induces a saturation in non-linear processes that characterize wave-structure interaction. Effect of saturation on 2nd order transmitted free waves amplitude was investigated, and a best-fit formula was derived. A simple approximate method for wave profile prediction leeward of the barrier is finally proposed.

1. INTRODUCTION
The use of submerged breakwaters for coastal protection is meeting, since last decades, a growing appreciation, mainly because of environmental and landscape matters. However, despite recent progresses, a certain lack of knowledge about hydraulic response of submerged structures can still be noticed, that can lead, in practice, to unexpected wave conditions and shoreline response in the sheltered area [10], [25].

In a 2D scheme, wave-structure interaction basically consists of two main processes:
1. wave energy dissipation caused by reflection, friction, and, possibly, breaking over the barrier;
2. modification in wave spectrum due to non-linear effects in the shallow water area above the breakwater.

A large body of literature exists on the first aspect, including mainly experimental studies and curve-fitting for prediction of the transmission coefficient, ratio between square roots of 0th moment of incident and transmitted wave spectra (see, among others, [4], [5], [8]).

Regarding the second matter, considering a monochromatic incident wave with frequency $f$, as wave propagates above the barrier, super-harmonic components with multiple frequencies $2f$, $3f$, etc. are generated, either as bound waves (i.e. with same celerity and initial phase as fundamental harmonic) or free waves (i.e. with their own initial phase and celerity, obeying dispersion equation). As wave train crosses the obstacle, both types of super-harmonic components are propagated in the sheltered zone, affecting the frequency distribution of transmitted wave energy. Harmonic generation was theoretically studied, for a rectangular impermeable step and non-breaking waves, using Stokes’ formulation at 2nd order approximation [13], [20]. Further investigations were conducted by other authors by means of both mathematical and physical models, mostly leaving wave breaking out of consideration [17], [19], [22], [23], [27].

In works dealing with breaking waves, results generally show a reduction of transmitted super-harmonic components in the sheltered area with respect to non-breaking cases [3], [14], [28], [29]. Thus, it can be argued that wave breaking would induce a saturation in nonlinear interaction, even if the mechanism is not clearly defined. In order to better understand this process, theoretical and experimental investigations are needed, including description of wave breaking mechanisms above the barrier, that can be quite different from those observed on slopes and natural beaches.

In this regard, little research is nowadays available on wave breaking features at submerged breakwaters. Most studies refer to smooth impermeable barriers with a simplified geometry, frequently rectangular [2], [21], [26]. As an exception, in [16], authors tested trapezoidal submerged breakwaters with different porosity and proposed empirical formulae for prediction of the limiting wave height to submergence ratio based on the so-called “modified surf-similarity parameter”, originally introduced for solitary waves [15]. Finally, experimental tests using trapezoidal barriers with different slopes were performed [18]; authors investigated breaking limit and reported the different breaker forms observed at the barrier.

In recent experimental works [6], [24] wave breaking macrofeatures at submerged rubble-mound breakwaters with different geometry and permeability were described and a tentative classification/parameterization were proposed, based on both hydrodynamic and morphological criteria. Same authors [7] investigated changes in frequency distribution of wave energy due to interaction with a submerged barrier for both non-breaking and breaking conditions. Results, consistently with early literature, show that, for non-breaking waves, transmitted wave spectrum tends to spread towards
higher frequencies as incident wave height or period grows. In case of breaking waves, transmitted super-harmonic components reduce progressively, for growing wave height, starting from the highest frequency component, that first attains saturation values. In the present work, results of an experimental study in wave flume at University of Napoli Federico II are reported. Research focused on different aspects of interaction between waves and submerged rubble-mound breakwaters, including amount of transmitted wave energy, harmonic generation at barrier, free and bound waves transmission at trailing side, characteristics of wave breaking over the structure. On the basis of video analysis, wave breaking features are described and different breaker forms are identified. Then, based on wave data collected at different positions, effects of wave breaking on super-harmonic components transmission are discussed, and an approximate method for predicting harmonic amplitudes and resulting wave profile in the sheltered area is finally proposed.

2. EXPERIMENTAL SETTING

Experimental tests were performed in the small wave flume at University of Napoli Federico II, Department of Hydraulic and Environmental Engineering “Girolamo Ippolito” (Fig. 1).

![Fig. 1. Wave flume used for tests](image1)

The flume is about 23.50m long, and is made of two different sections; the first, closer to wavemaker, has converging steel walls, and width varying from 0.80m at the wavemaker to 0.50m at a distance of 8.50m from it; the second section, with glass walls, is about 13.00m long, with a constant width of 0.50m. The height of flume is 0.75m for all its length. Waves are generated by a piston-type wavemaker, provided with dynamic wave paddle, provided with dynamic wave maker to 0.50m at a distance of 8.50m from it; different sections; the first, closer to wavemaker, has converging steel walls, and width varying from 0.80m at the wavemaker. The flume at University of Napoli Federico II, Department of Hydraulic and Environmental Engineering “Girolamo Ippolito” (Fig. 1).

![Fig. 2. Cross-section of tested breakwater](image2)

Regular waves were used for tests, with target wave heights at wavemaker ranging from 2.0cm to 12.0cm, with 1.0cm step, and three wave periods, namely 1.0s, 1.5s and 2.0s.

3. DATA COLLECTION AND ANALYSIS

Resistive twin-wire gauges were used to measure surface elevation in four positions, namely, offshore, at the centre of seaward slope face, at the centre of barrier crown, and in the lee side of the structure, outside the reforming area. Sample frequency was 25Hz for all tests. Collected data were analyzed in frequency domain using FFT technique. From spectral analysis the following parameter were derived:

- 0th moment of wave spectrum: \( m_0 \)
- 1st moment of wave spectrum: \( m_1 \)
- -1 moment of wave spectrum: \( m_{-1} \)
- peak period: \( T_p \)
- mean period: \( T_{0,1} = m_0 / m_1 \)
- mean period: \( T_{-1,0} = m_{-1} / m_0 \)

Wave energy content \( E_{i}^{(r)} \) associated to generic \( i \)th harmonic component was derived from the corresponding spectrum peak (Fig. 3). Then, the spectral significant amplitude \( a_i \) of \( i \)th harmonic was derived as square root of 0th moment \( m_0^{(i)} \):

\[
a_i = \sqrt{m_0^{(i)}}
\]

Finally, the relative power of \( i \)th harmonic were derived as ratio between 0th moments:

\[
P_i = \frac{m_{0,i}^{(r)}}{m_0}
\]

![Fig. 3. Example of collected wave data and spectrum leeward of the breakwater](image3)

For each test, further surface elevation data were collected at trailing side of the breakwater, using an array of 2 wave gauges at distance \( \Delta x = 25.0cm \). From 2 gauges array data, transmitted free and bound super-harmonic components were then separated from fundamental transmitted 1st order harmonic using Grue’s method, described in [14].

The method assumes that, for a regular incident wave propagating in x direction with frequency \( f \), surface elevation in the sheltered area can be described by the following function:
\[ \eta(x,t) = a_1 \cos(k_1 x - \omega_1 t + \phi_1) + \sum_{n=2}^\infty a_n \cos(n(k_1 x - \omega_n t + \phi_n)) + \sum_{n=1}^\infty a_n \cos(k_n x - n \omega_1 t + \phi_n) \]

where:

- \( a_1 \) and \( \phi_1 \) express, respectively, amplitude and initial phase of 1\textsuperscript{st} (fundamental) harmonic, with frequency \( \omega_1 \).
- \( a_{bn} \) expresses amplitude of \( n \text{th} \) bound wave harmonic, with frequency \( n \omega_1 \). Note that initial phase for bound waves is the same as fundamental harmonic.
- \( a_{fn} \) and \( \phi_n \) express, respectively, amplitude and initial phase of \( n \text{th} \) order free wave, with frequency \( n \omega_1 \).

Wave numbers \( k \) and \( k_n \) obey dispersion equation, that, assuming linear theory, can be written as:

\[
\omega^2 = g k \tanh(kh) \quad \text{(4)}
\]

\[
(n \omega_1)^2 = g k_n \tanh(k_nh) \quad \text{(5)}
\]

being \( \omega = 2\pi f \) the fundamental wave pulsation, \( g \) the gravitational acceleration and \( h \) the water depth.

By introducing the Fourier transform of free surface elevation as:

\[
\hat{\eta}(x) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \eta(x,t) \cdot e^{-i\omega_1 t} \, dt \quad n=1,2,...
\]

then, according to author, amplitudes of free and bound waves can be determined as follows by given \( \hat{\eta}(x,t) \) at two different locations, namely \( x_1 \) and \( x_1 + \Delta x \):

\[
a_{bn} = \frac{1}{\sin\left(\frac{1}{2} (k_n - nk) \Delta x\right)} \left| \hat{\eta}(x_1) - \hat{\eta}(x_1 + \Delta x) \cdot e^{i\omega_1 n \Delta x} \right| \quad \text{(7)}
\]

\[
a_{fn} = \frac{1}{\sin\left(\frac{1}{2} (k_n - nk) \Delta x\right)} \left| \hat{\eta}(x_1) - \hat{\eta}(x_1 + \Delta x) \cdot e^{i\omega_1 n \Delta x} \right| \quad \text{(8)}
\]

Video camera recordings were taken during tests using two digital cameras, one in fixed position, the other moving along the flume.

4. WAVE BREAKING MACROFEATURES OVER THE BREAKWATER

Video data analysis was performed in order to indicate, for each test, whether wave breaking over the breakwater occurred or not and, if yes, to describe morphological features of the process. In order to reduce the subjectivity of the analysis, observations were repeated more times and independently by different observers.

In the work hereby reported, a hydrodynamic-morphological criterion was used to discriminate different breaker types, as discussed in detail in [6] and [24]. The method is based on consideration that, in all cases, wave breaking at barrier was observed to initiate with a water jet (“plunging jet” hereafter) moving from the wave front and projected in the wave direction, which strikes the water surface onward. Breakers classification was then based on the part of wave front from which plunging jet detaches from wave profile and shape of water surface at incipient breaking. Observed breaker macrofeatures are here only briefly outlined, for more details, see [6] and [24].

For lower wave period (\( T=1.0s \)) breaker types evolve, for growing heights, from the typical “spilling” to “plunging” forms, as observed on slopes and beaches [11]. Intermediate forms (so-called “spilling-plunging” breakers) were observed. It is important to remark that, as discussed in [1], spilling-to-plunging breakings are basically part of the same hydraulic phenomenon. Main features of this breaker class (indicated as “SpP breakers”) are the plunging jet detaching from wave crest and skewness of wave profile at breaking, with steep onward face and rear slope descending in offshore direction (Fig. 4, 5).

For higher wave periods, a second breaker class was recognized, indicated as “bore” breakers. Here wave profile at incipient breaking is made of two parallel subhorizontal sketches at different levels, connected by a central turbulent air-water zone, coming from the impact on wave profile of plunging jet detaching from crest. Breaker profile is very similar to a bore or hydraulic jump. In some cases (observed for \( T=2.0s \)) the water profile remains substantially unchanged as breaker travels above the barrier (Fig. 6). In other cases (observed for \( T=1.5s \)) breaker profile steepens above the crown and evolves in a form similar to a spilling or plunging (Fig. 7); these latter cases were classified as “bore-SpP breakers”.

Observed breaker types are summarized in Table 1, where \( Ur \) is the Ursell number of incident wave, with
period \( T \) and height \( H \), that can be regarded as a measure of wave non-linearity:

\[
Ur = \frac{H L_0^2}{h^3}
\]

being \( L_0 \) the deepwater wavelength, function of wave period \( T: L_0 = \frac{g T^2}{2 \pi} \).

Table 1. Breaker types observed during tests

<table>
<thead>
<tr>
<th>Test no.</th>
<th>( T ) (s)</th>
<th>( H ) (cm)</th>
<th>( Ur )</th>
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</tr>
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</tr>
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<td>11.00</td>
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</table>

Legend. NO: non-breaking; S: spilling; S-P: spilling-plunging; P: plunging; B: bore; B-StP: bore-StP

5. BREAKER INDEX ESTIMATION

Transition, for growing wave height, from non-breaking to breaking conditions was investigated. In particular, the breaker index, i.e. the lower value of incident wave height \( (H_i) \) to submergence \( (R_c) \) ratio after which wave breaking was observed to occur, was studied as function of wave and structure parameters. The following Goda-type expression [12] was used to interpolate experimental data:

\[
\frac{H_i}{R_c} = A \left[ \frac{L_0}{R_c} \right]^{-0.5} \exp \left( -1.5 \frac{\pi R_c}{L_0} \right)
\]

The best-fit value for scale parameter \( A \) was empirically estimated as \( A = 0.203 \). Experimental data and Eq. (10) are plotted in Fig. 8.

A good agreement between experimental points and Eq. (10), for different values of scale parameter \( A \), was observed also for different barrier geometry and permeability, as reported in [6] and [24].

Fig. 8. Experimental data and breaker index

6. WAVE MODIFICATION DUE TO HARMONIC GENERATION OVER THE BARRIER

6.1. NON-BREAKING WAVES

As previously discussed, as wave propagates in the shallow water area above the breakwater, higher order harmonic components are generated. Super-harmonic components then propagate in the sheltered area, affecting transmitted wave energy frequency distribution.

Fig. 9 illustrates, for example purpose, the relative power of the first three harmonics at different positions for test no. 4, corresponding to a nearly linear incident wave. Wave-structure interaction is evident on the top of the barrier, where the power of fundamental harmonic shows a 20% reduction with respect to incident wave; higher order components power (especially at 2nd order) grows correspondingly, with a maximum on the breakwater crown. Wave power distribution shows only little changes as wave train propagates in deeper area behind the barrier.

As Ursell number of incident wave grows, wave modifications can be more complex from those indicated in Fig. 9. However, results generally indicate that, for non-breaking wave, wave-structure interaction induces energy transfer from 1st harmonic to multiple frequencies components. Keeping wave period constant, the process

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is more intense as wave height grows.

6.2. BREAKING WAVES

Experimental results with breaking waves over the barrier indicate, in general, a limitation in harmonic generation and wave energy transfer with respect to non-breaking cases.

Fig. 10 illustrates the relative power of harmonic components at different positions for test no. 9. Analogously to non-breaking cases, super-harmonic components grows above the barrier, causing a reduction in the relative power of fundamental harmonic. As wave breaks above the barrier, higher order components show a dramatic reduction, and tend to disappear in the sheltered area, so that resulting transmitted wave is nearly linear.

For higher incident wave periods, the process can be different from that showed in Fig. 9, as observed for non-breaking waves, and transmitted wave spectrum generally presents a significant amount of energy at higher frequencies. However, as incident wave height grows, after breaking, higher frequencies power seems to show a limitation in growth, tending to an asymptotic value, as reported in [28] and [29].

7. SUPER-HARMONIC SATURATION DUE TO WAVE BREAKING OVER THE BARRIER

Comparison among tests with non-breaking and breaking waves suggests, consistently with early literature, that wave breaking induces a saturation in harmonic generation due to wave-structure interaction.

As previously observed, effect of saturation on transmitted wave field can significantly vary from one case to another. This is probably due to superposition, in the sheltered area, of super-harmonic components ascribable to two different non-linear processes:

1. harmonic generation induced by wave-barrier interaction;
2. non-linearity due to limited depth behind the barrier, depending on wave height and period, analogously to what can be observed for incident wave.

First class of super-harmonics can be identified with transmitted free waves, whilst the second with transmitted bound waves. The contributions of two classes cannot be discriminated by a simple frequency domain analysis.

In the view of a better understanding of the breaking-induced saturation process, transmitted fundamental and higher order free wave amplitudes leeward of breakwater were derived using 2 gauges array data and Grue’s method, as described above. For sake of simplicity, super-harmonic analysis were limited to 2nd order components, i.e. components with larger energy content.

In Fig. 11 the ratio \( \frac{a_2}{a_1} \) (where \( a_2 = \) transmitted 2nd order free wave amplitude and \( a_1 = \) transmitted 1st harmonic amplitude) is plotted for different values of incident wave amplitude. Results are compared with Grue’s experimental data [14] in the lee of a rectangular barrier with base 50.0cm and submergence \( R_c = 5.0cm \).

All graphs show similar trends. For lower values of incident wave amplitude, the ratio \( \frac{a_2}{a_1} \) grows as \( a \) grows, due to stronger non-linear processes above the barrier, with maximum value approximately corresponding to initial breaking conditions; then, \( \frac{a_2}{a_1} \) decreases as incident wave amplitude increases, showing the effect of saturation process induced by wave breaking.

In order to emphasize the rule of breaking in harmonic saturation, the following non-dimensional parameter was introduced, as a breaking dissipation index \( N_{fb} \):

\[
N_{fb} = \frac{H_i}{L_0} \left( \frac{B_{eq}}{R_c} \right)^{2/5}
\]  

where:
- \( H_i \) is the incident wave height;
- \( B_{eq} \) is the “equivalent width” of the barrier, ratio between area and height of trapezoidal section: for present breakwater \( B_{eq} = 68.75cm \);
- \( R_c \) is the freeboard over the top of the barrier;
- \( L_0 \) is the deepwater length.

The above introduced parameter can efficiently describe energy dissipation due to wave breaking, that, for unitary width, is directly proportional to energy flux and inversely proportional to water depth [9].

In Fig. 12 the ratios \( \frac{a_2}{a_1} \) are plotted against the breaking dissipation index above defined. The effect of breaking can be clearly observed for higher values of \( N_{fb} \), i.e., for a given data series, for higher incident wave heights.
Considering only the values of $a_2/a_1$ showing a descending trend with $N_b$, experimental data can be fitted reasonably well by the following logarithmic function:

$$\frac{a_2}{a_1} = -0.21 \ln(N_b) - 0.89$$  \hspace{1cm} (12)

The correlation coefficient is good: $R^2=0.96$.

From what stated above, Eq. 12 can be regarded as a “saturation function”, expressing the transmitted 2nd order free wave amplitudes as a function of incident wave and structure parameter, in the cases of wave breaking over the barrier.

### 8. ESTIMATION OF 1st ORDER TRANSMITTED WAVE AMPLITUDE

In Fig. 13 the ratio between measured transmitted 1st harmonic $(a_1)$ and incident wave amplitude $(a)$ is plotted against non-dimensional freeboard $R_f/H_i$, being $H_i$ the incident wave height. Consistently with conventional notation for submerged barrier, freeboard $R_f$ is considered as a negative number.

The ratio $a_1/a$ represents 1st order transmission coefficient $K_{1t}$. Experimental results shown in Fig. 13 are compared with values predicted using the well known transmission formula proposed by d’Angremond et al. [8]:

$$K_{1t} = -0.40 \frac{R_f}{H_i} + 0.54 \left( \frac{B}{H_i} \right)^{-0.31}$$  \hspace{1cm} (13)

with limitation $K_{1t} \leq 0.80$.

Despite a certain scatter, a reasonable agreement among measured and predicted data can be observed. Similar results were also obtained using different formulation. Thus, it can be argued that traditional transmission formulae can be efficiently used to predict 1st order wave transmission in the lee of the barrier.

Result is not surprising, considering that traditional empirical formulae for wave transmission, even when dealing with random waves, are substantially based on a linear approach.

### 9. WAVE PROFILE PREDICTION LEEWARD OF BREAKWATER

From results above discussed, a simple approximate method for predicting wave profile in the sheltered area can be proposed.

Neglecting higher order components, transmitted wave profile can be described by the following function:

$$\eta(x,t) = a_1 \cos(k x - \omega t + \phi_1) + a_{2c} \cos(2(k x - \omega t + \phi_2) + a_{2b} \sin(2(k x - \omega t + \phi_3))$$  \hspace{1cm} (14)

First term is the fundamental transmitted harmonic, and its amplitude $a_1$ can be estimated from incident wave and structure parameters using an empirical wave transmission formula (e.g. Eq. 13). Second term is the 2nd order transmitted bound wave. Its amplitude basically depends on water depth behind the barrier $h$, 1st order wave amplitude $a_1$, and period $T$, and can be estimated using Stokes’ theory:

$$a_{2b} = k a_1^2 \cosh(k h) \left[ 2 + \cosh(2kh) \right]$$  \hspace{1cm} (15)

Third term in Eq. (14) is the 2nd order transmitted free wave. Amplitude can be estimated, for breaking wave over the barrier, assuming the saturation value given by Eq. (12).

Wave numbers $k$ and $k_2$ are calculated using dispersion equation, see Eq. (4) and Eq. (5).

It is noteworthy that, whilst 2nd order bound wave propagates with the same initial phase $\phi_2$ as fundamental harmonic, initial phase $\phi_3$ is different for 2nd order free wave term; initial phase shift cannot be predicted using the proposed method.

On the other hand, from Eq. (14), it can be observed that, for a given time $t$, phase shift between harmonic components can significantly vary with abscissa $x$ due to difference in wave number of 2nd order free wave with

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**Fig. 12.** Ratio between transmitted 2nd order free wave and 1st harmonic amplitudes, plotted against incident breaking dissipation index.

**Fig. 13.** 1st order transmission coefficient

**Fig. 14.** Comparison between measured and predicted wave profile behind the barrier.
respect to the other components. Consequently, non-
permanent wave forms are to be expected leeward of the
barrier, with a resulting profile significantly varying as
wave train propagates in protected area.
However, despite limitations due to introduced
approximations, comparison between experimental data
and wave profile predicted using Eq. (14) shows,
generally, a reasonable agreement, for different incident
wave conditions.
For example purpose, profiles shown in Fig. 14 refer to
test no. 18, with incident wave height $H=9.0$cm and
period $T=1.5$s. Regardless of inevitable phase shift, wave
forms are substantially coincident, indicating that
proposed method can be considered sufficiently realistic
in predicting wave profile leeward of the breakwater.

10. CONCLUSIONS
Results hereby reported show, in general, a good
agreement with early theoretical and experimental studies.
Observed breaker types above the barrier can be
comprised in two main classes. First class, corresponding
to lower incident wave period, includes classical spilling-
to-plunging breakers, very similar to those observed on
slopes and natural beaches. Second class comprises
breaker types macroscopically similar to a bore, or
hydraulic jump, that are to be considered typical of
submerged barriers.
Effect of wave breaking on non-linear wave-structure
interaction was investigated. Results show that as wave
breaks over the dike, transmitted 2nd order free wave
amplitudes progressively reduce as intensity of energy
dissipation increases; experimental data can be adapted
to a best-fit logarithmic expression, that can be regarded as a “saturation formula” for harmonic generation.
Moreover, results show that transmitted 1st order wave
amplitude can be reasonably well described by empirical
transmission formulae present in literature.
Finally, a simple approximate method for predicting
transmitted wave profile is proposed, for waves breaking
over the structure. Comparison among measured and
predicted wave forms shows a reasonable agreement.

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