ON THE USE OF U-TYPE STABILIZING TANKS FOR REDUCING ROLL MOTIONS IN HEAD SEAS

Marcelo A. S. Neves, LabOceano - COPPE/UFRJ
Jorge A. Merino, COPPE/UFRJ
Claudio A. Rodriguez, LabOceano - COPPE/UFRJ
Luiz Felipe N. Soares - UFRJ

SUMMARY
Many investigations have been conducted on the use of U tanks in the control of the roll motions in beam seas. However, very few papers have dealt with their application in the case of parametric resonance in longitudinal waves. The present paper describes the development of a mathematical model in which the fluid motion inside a U tank is non-linearly coupled to the heave, roll and pitch motions of the ship.

A transom stern small vessel, a vessel well known to be quite prone to parametric amplification is employed in the study. The results are presented in the form of limits of stability, with encounter frequency and wave amplitudes as parameters. Distinct dynamical characteristics are discussed and conclusions are drawn on the relevant parameters for the efficient control of the roll amplifications in head seas.

1. INTRODUCTION
In longitudinal waves a ship may be forced to roll heavily. This is not due to direct wave excitation. Instead, an internal excitation is the mechanism responsible for the so-called parametric excitation. In face of some serious recent incidents of great roll amplification in head seas, as reported in [1], great attention has been given recently to head seas parametric resonance, see for example refs. [3, 6, 11], and possible control of head seas parametric rolling.

Refs. [2, 4, 10, 12, 14, 15] have discussed in detail the application of anti-rolling devices and particularly anti-rolling tanks (ART) to the reduction of roll motions of ships subjected to external wave excitations.

The present paper summarises the development of a non-linear mathematical model of ART dynamics derived in [5] compatible with the 3rd order mathematical model introduced by Neves and Rodriguez [7, 8] for the analysis of parametric rolling in head seas. In the present model the fluid motion inside a U tank is non-linearly coupled to the heave, roll and pitch motions of the ship. A transom stern small vessel, a vessel well known to be quite prone to parametric amplification is employed in the study. The results are presented in the form of limits of stability, with encounter frequency and wave amplitudes as parameters. Distinct dynamical characteristics are discussed and conclusions are drawn on the relevant parameters for the efficient control of the roll amplifications in head seas.

2. COORDINATE SYSTEMS AND TANK DEFINITIONS
In order to describe the ship motions and internal fluid motion two reference axes are employed. An inertial system CXYZ moving at the average ship speed (U), such that at instant t=0 the plane XY coincides with the calm water free surface with point C at the same vertical of the centre of gravity G of the ship. A second coordinate system is the moving frame Oxyz. This second system is fixed to the ship such that the plane XY coincides initially with the ship waterplane in calm water and axis Ox belongs to the diametral plane, being positive in the sense of ship speed, axis Oy points to portside and Oz axis passes always by the vertical line containing the ship centre of gravity G, positive upward. The two reference frames are shown in Figure 1.

[Figure 1 – Inertial and fixed coordinate axes]

Geometric characteristics of the U tank are introduced in Figure 2. Two vertical reservoirs are connected by a transversal duct, with all elements having rectangular constant cross sections. At the centre of the duct a pump is contemplated as a possible device to control the fluid motion. Fluid motion, assumed to be unidirectional, is described by fluid displacement Z(t) and relative fluid velocity Û(t). It should be noted that point O not
necessarily coincides with \( \mathbf{G} \), the ship centre of gravity, assumed to be positioned (unmovable) at \((0,0,z_G)\). In later developments angular variable \( \tau(t) = \tan^{-1} \left( \frac{Z(t)}{B_W} \right) \) will be employed instead of the linear variable \( Z(t) \). Its geometric interpretation is made clear by noting Figure 3.

\[
\begin{bmatrix}
W_B \\
Z(t) \\
\end{bmatrix} + \begin{bmatrix}
B_W \\
\end{bmatrix} = \begin{bmatrix}
\zeta \\
\phi \\
\end{bmatrix} = 0
\]

(1)

where vector \( \mathbf{s}(t) \) represents rigid body motions in six degrees of freedom. In equation (1) \( \mathbf{M} \) is a 6x6 matrix which describes hull inertia characteristics. Its elements are: \( m \), the ship mass, first moment \( m z_G \), \( J_{xy} \) and \( J_{xz} \) the mass moments of inertia in the roll, pitch and yaw modes, respectively, and \( J_{az} \), the roll-yaw product of inertia, all moments taken with reference to the chosen origin. \( \mathbf{A} \) is also a 6x6 matrix, whose elements represent hydrodynamic generalized added masses. \( \mathbf{B}(\phi) \) describes the coefficients of the hydrodynamic reactions dependent on ship velocities (damping), and may incorporate non-linear terms in the roll equation. \( \mathbf{C}_r(\mathbf{s},\mathbf{\zeta}) \) is a 6x1 vector which describes non-linear restoring forces and moments dependent on the relative motions between ship hull and wave elevation \( \zeta(t) \). On the right hand side of equation (1), the generalized vector \( \mathbf{C}_{ext}(\zeta,\phi) \) represents wave external excitation, usually referred to in the literature as the Froude-Krilov plus diffraction wave forcing terms, dependent on wave heading \( \chi \), encounter frequency \( \omega_e \), wave amplitude \( A_w \) and time \( t \). Finally, generalized vector \( \mathbf{C}_r(Z,\dot{Z},\ddot{Z},\dot{s},\ddot{s},\ddot{s}) \) represents forces and moments acting on the ship due to the fluid motion inside the tank. For a ship without tank, equation (1) reproduces the system of non-linear equations introduced in Refs. [7, 8, 9].

4. DERIVATION OF THE ART EFFECTS

Under the assumption of unidirectional fluid motion, the differential momentum equations of internal fluid motion may be somewhat simplified by considering the dynamics of elementary fluid volumes. Figure 4 illustrates the free body diagram of an elementary fluid volume considered at the portside reservoir. Similar free body diagrams are defined for the left and right elements of the duct and starboard reservoir. At each element, the momentum rate is defined as:

\[
dF = dF_w + dF_{wall} + dF_p = \rho A a dh
\]

(2)

where \( \rho \) is the internal fluid density, \( A \) is the cross sectional area of the reservoir, \( a \) is the absolute acceleration of the elementary volume considered and \( dh \) is an increment defined along the reservoir axis, as indicated in Figure 2. It is pointed out that the mass of an elementary volume is \( dm = \rho A dh \).

5. DERIVATION OF FLUID ACTIONS ON THE PORTSIDE RESERVOIR

In the following, expressions for all the terms in equation (2) will be detailed for the portside reservoir.

a) The elementary weight is defined as:
Figure 4 – Elementary fluid volume: port side reservoir.

\[
d\mathbf{F}_w = -\rho g A \hat{g}_u d\mathbf{h} = \rho g A (\sin \theta_i + \cos \phi \cos \theta j) + \cos \phi \cos \theta k) d\mathbf{h}
\]

where \( \hat{g}_u = -\sin \theta_i + \sin \phi \cos \theta j + \cos \phi \cos \theta k \) is the unit vector along the vertical.

b) The force on the wall, as indicated in Figure 4, is expressed as:

\[
d\mathbf{F}_{wall} = dN_i \mathbf{i} + dN_j \mathbf{j} - \tau \mathbf{P} e \mathbf{d} \mathbf{h} \mathbf{k}
\]

where:

\( \tau \) is the surface tension on the walls of the reservoir,

\( \mathbf{P} e \) is the perimeter of the cross section of the reservoir and

\( N_i, N_j \) are the longitudinal and transversal components on the walls.

c) Net pressure force is expressed as (see Figure 4):

\[
d\mathbf{P}_n = A d \mathbf{P} \mathbf{k}
\]

d) Elementary volume:

\[
dv = A \mathbf{d} \mathbf{P} \mathbf{k}
\]

e) Absolute acceleration:

\[
a = a_0 + \Omega \times (\Omega \times r_B) + \ddot{\Omega} \times r_B + 2\Omega \times \mathbf{v}_B + \mathbf{a}_B
\]

where:

\( \mathbf{v}_B = \dot{Z}(t) \mathbf{k} \)

\( \mathbf{a}_B = \ddot{Z}(t) \mathbf{k} \)

are the relative velocity and acceleration, respectively. Additionally,

\[
r_B = L_i + B \mathbf{j} - (H_2 + h) \mathbf{k}
\]

where \( L_i \) is the longitudinal position of the tank expressed in the ship coordinate system.

\[
a_0 = \frac{d\mathbf{v}_0}{dt} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} + \Omega \times \mathbf{v}_0
\]

\[
v_0 = \frac{d\mathbf{v}_0}{dt} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}
\]

\( \Omega = pi + qj + rk \)

\( \Omega = pi + qj + rk \)

Substitution of equations (3.1 ~ 3.11) into equation (2) will result in explicit expressions for the unknown differentials \( dN_i, dN_j, \) and \( d\mathbf{P} \) for the portside reservoir. Subsequently integrating from \( h = Z \) to \( h = H_2 \), expressions for forces acting on the portside reservoir are obtained as:

\[
N_n = \rho A, \{[-\sin \theta + \hat{u} + qw - rv - (q^2 + r^2)L_i - (r - pq)B_a + 2q\hat{Z}]H_i - Z) - \frac{1}{2}[(\hat{q} + pr)[L_i^2 - (H_2 + Z)^2]]
\]

\[
N_h = \rho A, \{[\sin \theta \cos \phi + \hat{v} + ru - pv - (p^2 + r^2)B_a + (r - pq)L_i - 2p\hat{Z}]H_i - Z) + \frac{1}{2}[(\hat{p} - qr)[L_i^2 - (H_2 + Z)^2]]
\]

\[
P_p - P_i = -\rho [\frac{\tau \mathbf{P} e \mathbf{d} \mathbf{h} \mathbf{k}}{\rho A} + g \cos \phi \cos \theta + \hat{w} + \hat{v} - qu - (q - pr)L_i + (\hat{p} - qr)L_i + \frac{E}{2\rho A \hat{Z}} + (g \cos \phi \cos \theta + \hat{w} + \hat{v} - qu - (q - pr)L_i + (p^2 + q^2)H_i)Z]
\]

\[
\tau_{rd} = \frac{P_p - P_i}{2\rho} - [\sin \phi \cos \theta + \hat{v} + ru - pv - (r - pq)L_i + (p - qr)H_i]B_a
\]

Equivalent derivations have been developed for the other sections of the tank. Complete details may be found in [5]. Integrating the forces and moments along the whole length of the tank the non-linear equation of fluid motion may be obtained:

\[
W_i (\frac{B_a}{H_i} \hat{Z} + \frac{Pe \mathbf{P} e B_a}{\rho A} \mathbf{r}_a) - \frac{Pe \mathbf{P} e H_i}{\rho A} \mathbf{r}_a + KH_2Z - \frac{E}{2\rho A \hat{Z}} + (g \cos \phi \cos \theta + \hat{v} + ru - pv - (q - pr)L_i + (p^2 + q^2)H_i)Z
\]

where \( E \) is the power delivered to the fluid by a pump and \( K \) is a localized head loss. In equation (5) the tangential tensions on the walls \( (\tau_r, \tau_d) \) may be expressed as [13]:

\[
\tau_{rd} = f \gamma \hat{Z}^2 \frac{8g}{8g}
\]
where $f$ is a friction factor, assumed to be dependent on the Reynolds number:

$$Re = \frac{\dot{Z}D_{r,d}}{v}$$

The characteristic length $D_{r,d}$ shall be the reservoir width ($W_r$) or the duct height ($H_d$) depending on what part of the tank the Reynolds number is computed. The friction factor may be estimated by means of semi-empirical formulas found in the literature, [13].

As a further approximation, it will be assumed that the damping actions are proportional to fluid velocity $\dot{Z}$, such that an equivalent damping ($B_{\tau\tau}$) will be defined as:

$$\frac{Pe_d}{\rho A_j}B_{\tau\tau} = \frac{Pe_d}{\rho A_j}H_r \tau_r + K\dot{Z}^2 \approx B_{\tau\tau} \dot{Z}$$

Internal damping coefficient $B_{\tau\tau}$ may be determined by means of decay tests:

$$\eta_i = \frac{B_{\tau\tau}}{2 \sqrt{g W_r (\frac{B_w}{H_d} - \frac{H_r}{W_r})}}$$

As additional simplifications it will be assumed that the pump is not operating ($E=0$) and the portside and starboard pressures are equal to the atmospheric pressure ($P_p = P_a = 0$), such that the tank is then in a purely passive mode of operation.

### 6. FORCES AND MOMENTS EXERTED BY THE TANK ON THE SHIP

Tank force for an elementary volume of fluid is expressed as:

$$dF_{\text{tank}} = -(a_A + gg_{\text{in}}) dm$$

and the corresponding moment as:

$$dM_{\text{tank}} = -r_B \times (a_A + gg_{\text{in}}) dm$$

such that the generalized vector $\vec{C}_i \begin{pmatrix} Z, \dot{Z}, \ddot{Z}, \dddot{Z}, \dot{s}, \ddot{s}, \dddot{s} \end{pmatrix}$, defined in equation (1), may be separated in:

$$\vec{C}_i = \begin{bmatrix} C_{11} & \cdots & C_{16} \\ C_{21} & \cdots & C_{26} \\ \vdots & \ddots & \vdots \\ C_{16} & \cdots & C_{16} \end{bmatrix} = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} \begin{bmatrix} C_{14} \\ \cdots \\ C_{16} \end{bmatrix} \begin{bmatrix} \dot{K}_a \\ \cdots \\ \dot{N}_a \end{bmatrix}$$

(6)

with the following expressions:

$$X_a = -\rho A_j \begin{pmatrix} \rho g \sin \theta + \dot{u} + qw - rv - L_r(q^2 + r^2) + 2B_w(\dot{r} - pq)Z - C_i(q + pr) - 4(A(\dot{r} + qZ)) \end{pmatrix}$$

(7-1)

$$Y_a = -\rho A_j \begin{pmatrix} \rho \sin \theta \cos \theta + \dot{v} + ru - pw + L_n(\dot{r} + pq) + 4pZ + C_i(p - qr) + 2B_w(p^2 + r^2) + 2B_w \dot{Z} \end{pmatrix}$$

(7-2)

$$Z_a = -\rho A_j \begin{pmatrix} \rho \cos \theta \sin \theta + \dot{w} + pv - qu - \dot{L}_r(q - pr) + 4B_w \dot{Z} + C_i(p^2 + q^2) - 2B_w(p + qr)Z - 2ZZ \end{pmatrix}$$

(7-3)

$$K_a = -\rho A_j \begin{pmatrix} 2B_w(\dot{H}_r + L_n) + 4H_pW + 1/3B_z^2(C_i + 4H_j)(\dot{r} + pq) \end{pmatrix}$$

(7-4)

$$M_a = \begin{pmatrix} 2L_z + 4(H_rqZ + B_wL_r - B_rL_p)Z - \dot{L}_rC_i \begin{pmatrix} \rho \cos \theta \cos \theta + \dot{w} - pv - qu - L_n(\dot{r} - pq) \end{pmatrix} + 2/3(3B_xL_z + 3H_2H_r - 3H_1H_r^2 + 3H_0H_r^2 + H_1^3)(\dot{r} + pq) + 2B_w(\dot{r} + pq)Z + 2B_wL_n(\dot{r} + pq)Z - C_i[\rho \sin \theta + \dot{u} - rv + qw + L_n(\dot{r} + pq)] \end{pmatrix}$$

(7-5)

$$N_a = \begin{pmatrix} 2L_y \begin{pmatrix} B_w + 4(L_rqZ - B_wH_r) \end{pmatrix} + 2B_x[\rho \sin \theta \cos \theta + \dot{v} - ru - pq + \dot{L}_r(p^2 - q^2) - H_r(q + pr)Z + C_iL_n(\dot{r} + pq) + 2C_iL_n(\rho \sin \theta + \dot{v} + ru - pw + L_n(\dot{r} + pq)] + 2B_wL_n(\dot{r} + pq) + 2B_wL_n(\dot{r} + pq) \end{pmatrix}$$

(7-6)

where:

$$\vec{B}_w = \frac{B_w}{R} \begin{pmatrix} 2(H_r + \vec{B}_w) \end{pmatrix}, \quad C_1 = 2(H_r + \vec{B}_w),$$

$$C_2 = L_z^2 - H_z^2 - Z^2 + 2L_z \vec{B}_w$$

and $\vec{T}$ is the 3-D matrix of rotations.

Thus, equation (1), with the last term on the right hand side given by equations (7), together with equation (4) will form a set of 7 non-linear coupled second order differential equations describing the six degrees of freedom ship motions and fluid motion.

### 7. PARAMETRIC RESONANCE IN LONGITUDINAL WAVES

Focusing on the problem of parametric rolling in longitudinal waves, the dynamic coupled ship/tank problem may be investigated with reference being made to the heave-roll-pitch-tank problem, resulting in a four
degrees of freedom problem. Neglecting surge, sway and yaw motions \( (X_{sa} = Y_{sa} = N_{sa} = 0) \), and redefining the coefficients into a derivative nomenclature introduced in Ref. [5], results in the following four equations:

\[
Z_{sa} = Z_{s,ao} \ddot{z} + Z_{s,ao} \dot{\phi} \dot{t} + Z_{sp} \phi \dot{t} + Z_{sa} \phi^2 + + Z_{s} \cos \phi \cos \theta + + Z_{s} \dot{\phi} \dot{r} + Z_{s} \phi \phi^2 + \tag{8-1}
\]

\[
K_{sa} = K_{s,ao} \ddot{r} + K_{s,ao} \phi \theta + K_{s,ao} \phi \theta + + K_{s} \phi \dot{\phi} + K_{s} \phi \dot{\phi} + + K_{s} \phi \phi^2 + + K_{s} \phi \phi^2 + \tag{8-2}
\]

\[
M_{sa} = M_{s,ao} \ddot{z} + M_{s,ao} \phi \theta + M_{s,ao} \sin \phi \cos \theta + + M_{s,ao} \ddot{r} + M_{s,ao} \phi \dot{\phi} + + M_{s,ao} \phi \dot{\phi} + + M_{s,ao} \phi \phi^2 + + M_{s,ao} \phi \phi^2 + \tag{8-3}
\]

\[
T_{s,ao} \ddot{t} + T_{s,ao} \phi \theta + T_{s,ao} \phi \theta + + T_{s} \phi \dot{\phi} + T_{s} \phi \dot{\phi} + + T_{s} \phi \phi^2 + + T_{s} \phi \phi^2 + \tag{8-4}
\]

The new derivative coefficients are shown in Table 1 in terms of the tank parameters. The set of four non-linear ordinary differential equations may be numerically solved for a given vessel and a specified tank positioned at some defined position on the vessel. It may be noticed that in the particular case of roll/tank linear equations the problem simplifies to:

\[
K_{sa} = K_{s,ao} \ddot{r} + K_{s,ao} \phi \theta + K_{s,ao} \phi \theta + + K_{s} \phi \dot{\phi} + K_{s} \phi \dot{\phi} + + K_{s} \phi \phi^2 + + K_{s} \phi \phi^2 + \tag{8-2}
\]

\[
T_{s,ao} \ddot{t} + T_{s,ao} \phi \theta + T_{s,ao} \phi \theta + + T_{s} \phi \dot{\phi} + T_{s} \phi \dot{\phi} + + T_{s} \phi \phi^2 + + T_{s} \phi \phi^2 + \tag{8-4}
\]

8. RESULTS

ART main dimensions are given in Table 1. Five values have been defined for the parameter \( H_d \) in order to obtain five distinct ratios for the tuning \( \omega_l / \omega_{as} \). The tank horizontal dimension \( L_t \) was chosen to be the free parameter to allow variations in the ratio of mass of internal water to ship mass \( m_i / m \). In order to investigate the general trends of the influence of the ART parameters, systematic simulations have been performed in the case of a fishing vessel denominated TS, described in [7]. Figure 5(upper) shows the heave, roll and pitch time series corresponding to conditions \( GM = 0.37 m, \quad F_n = 0.30, \quad \omega_l = 2.1 \omega_{as} \) for which intense parametric amplification occurred for the ship without tank. These conditions are compared in the lower graph with situations of reduction of the roll motion due to the tank. It is also observed that no significant changes in the vertical motions are detected. The complete cancellation of parametric rolling may be seen to be true even for extreme cases like in Figure 6. This is a condition in which the ship, without tank, would encounter capsizal. By application of a tank in heavy condition, even not being closely tuned, the roll motion completely disappears.

A question arises as to what combinations of wave amplitude and wave length the tank may induce beneficial reduction or elimination of the undesirable parametric rolling. Given the broad range of relevant parameters involved, it becomes necessary to map the areas of stable and unstable motions. Figure 7 shows the results for the limits of stability corresponding to ship TS without tank, for two speeds. It is observed that the area of unstable motions is large in both speeds. Clearly, the condition \( F_n = 0.30 \) corresponds to more intense amplifications, what had already been demonstrated by Neves and Rodriguez [8]. Here we have chosen to perform the mappings for this ship forward speed with stronger roll amplification, in order to reveal the whole spectrum of possible responses. In Figures 8 and 9 the limits of stability are presented for the ship with tank for a condition with 3% of mass of water with respect to the ship mass, and damping levels of 0.2 and 0.3, respectively, and different tank tunings, \( \omega_l / \omega_{as} \). It may be observed that in the cases of \( \omega_l / \omega_{as} = 1.0 \) and \( \omega_l / \omega_{as} = 1.1 \) better results are obtained, in the sense that the area of unstable motions substantially reduced and the unstable roll motions have small amplitudes.

Figures 10 and 11 show the effects on the limits of stability due to changes in the ratio of mass of the tank fluid to ship mass considering the tuning \( \omega_l / \omega_{as} \) and two levels of damping ratio. It may be observed that a ratio of \( m_i = 2m \) or more is required for the unstable areas to shrink. Unfortunately, that can only be achieved with a price to be paid in terms of deadweight. Residual roll amplification are encountered in the range of higher frequencies. As the mass \( m_i \) increases, the unstable area moves slightly to the right and markedly upwards, which is the relevant effect to be expected in the design of an efficient ART. In Figure 11 the same mass variation is simulated, but now with a higher value for the internal damping. It is observed that in this
condition more substantial reductions in the area of unstable motions are found. Only at very high frequencies some amplification persists.

It is further observed that no significant roll amplification was observed inside the area of fundamental resonance for any set of tank parameters. Further research are expected to be conducted on this very extensive subject, particularly involving an experimental test program.

9. CONCLUSIONS

Non-linear equations for the coupled motions of a ship and the water inside a tank have been introduced. The set of equations is defined to the third order, compatible with the set of equations previously employed by Neves and Rodriguez [7, 8]. In general the simulations have indicated that it is possible to control strong parametric resonance if the correct choice of parameters is made in the first region of instability. Mapping of stable and unstable regions have been obtained for different parameters of the ART. It has been shown that tuning of the tank and ship natural frequencies and internal mass are the crucial parameters for the successful utilization of an U-tank in reducing the risk of inception of parametric resonance. According to the results, internal damping plays a less marked influence. These are aspects that should be examined in more detail in the future. Numerical limits of stability may be seen to be an efficient tool in order to get a comprehensive assessment of the ART characteristics. Important to say, it is shown that for the range of parameters simulated, an ART may eliminate roll amplification at some conditions but persists (or appears) at some others. It is expected that model tests will be of great help in the validation of the present numerical simulations.

ACKNOWLEDGEMENTS

The present investigation is supported by CNPq within the STAB project (Non-Linear Stability of Ships), CAPES and LabOceano.

REFERENCES

### Table 1a: Heave/tank coefficients

<table>
<thead>
<tr>
<th>Z_{zz} = \rho_A C_1</th>
<th>Z_{r\theta} = -\rho_A L_z C_1</th>
<th>Z_{r\xi} = -2\rho_A B_w^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_{\phi r} = -2\rho_A B_w^2</td>
<td>Z_{\phi \theta} = 4\rho_A B_w^2</td>
<td>Z_{\phi \xi} = \rho_A (L_z^2 - H_z^2 + 2L_z B_w)</td>
</tr>
<tr>
<td>Z_{\theta \phi} = \rho_A (L_z^2 - H_z^2 + 2L_z B_w)</td>
<td>Z_{\theta \xi} = \rho_A C_1 g</td>
<td></td>
</tr>
</tbody>
</table>

### Table 1b: Roll/tank coefficients

\[
K_z = 2\rho_A B_w^2 (L_z + H_r)
\]
\[
K_{r\phi} = 2\rho_A B_w^2 H_2 + H_z^2 H_r + H_w H_z^2 + H_r^3 / 3 + B_w^2 (C_1 + 4H_r) / 6
\]
\[
K_{r\xi} = -2\rho_A B_w^2 g
\]
\[
K_{\phi \theta r} = -2\rho_A B_w^2 H_2
\]
\[
K_{\phi \xi r} = -\rho_A B_w^2 L_x
\]

### Table 1c: Pitch/tank coefficients

\[
M_{zz} = -\rho_A L_z C_1
\]
\[
M_{r\theta} = \rho_A g (L_z^2 - H_z^2 + 2L_z B_w)
\]
\[
M_{r\xi} = 2\rho_A (\bar{B}_w L_z^2 + H_z^2 H_r + H_w H_z^2 + H_r^3 / 3 + L_z^2 C_1 / 2)
\]
\[
M_{\phi r} = -4\rho_A B_w^2 L_x
\]
\[
M_{\phi \theta r} = -\rho_A (L_z^2 - H_z^2 + 2L_z B_w)
\]
\[
M_{\phi \xi r} = \rho_A B_w^2
\]
\[
M_{\theta \phi r} = 2\rho_A B_w^2 L_x
\]
\[
M_{\theta \xi r} = 2\rho_A B_w^2 L_x
\]
\[
M_{\phi \theta \xi} = \rho_A B_w^2
\]

### Table 1d: Tank coefficients

\[
T_z = 2\rho_A B_w^2 W_r \left( \frac{B_w}{H_z} \right)
\]
\[
T_z = 2\rho_A B_w^2 B_{tw}
\]
\[
T_\phi = 2\rho_A B_w^2 (L_z - H_r)
\]
\[
T_{rz} = 2\rho_A B_w^2
\]
\[
T_{r\theta} = -2\rho_A B_w^2 L_x
\]
\[
T_{r\xi} = -2\rho_A B_w^2 H_2
\]
\[
T_{\phi z} = 2\rho_A B_w^2 L_x
\]
\[
T_{\phi \theta} = 2\rho_A B_w^2 H_2
\]
\[
T_{\phi \xi} = 2\rho_A B_w^2 H_2
\]

### Table 2: ART main dimensions

<table>
<thead>
<tr>
<th>B_w (m)</th>
<th>W_r (m)</th>
<th>H_z (m)</th>
<th>H_d (m)</th>
<th>\omega_z (rd/s)</th>
<th>\omega_z / \omega_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 3.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.251</td>
<td>0.773</td>
<td>0.900</td>
</tr>
<tr>
<td>2 3.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.304</td>
<td>0.859</td>
<td>1.000</td>
</tr>
<tr>
<td>3 3.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.360</td>
<td>0.944</td>
<td>1.100</td>
</tr>
<tr>
<td>4 3.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.419</td>
<td>1.030</td>
<td>1.200</td>
</tr>
<tr>
<td>5 3.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.480</td>
<td>1.116</td>
<td>1.300</td>
</tr>
</tbody>
</table>
Figure 5 – Time series for ship TS, heave, roll, pitch, $GM=0.37m$, $Fn=0.20$, $Aw=0.6m$. Upper graph, without tank. Lower graph, with tank ($\omega_t = 1.5\omega_{n4}$, $m_i = 3%m$, $\eta_i = 0.3$).

Figure 6 – Roll motion for ship TS, $GM=0.37m$, $Fn=0.30$, $\omega_t = 2.45\omega_{n4}$, $Aw=0.7m$. Without tank, capsize condition; with tank, roll disappears ($\omega_t = 1.5\omega_{n4}$, $m_i = 3%m$, $\eta_i = 0.3$).

Figure 7 – Ship TS without tank. a) $Fn=0.20$; b) $Fn=0.30$. 

Session A
Figure 8 – Ship TS, $GM=0.37$, $Fn=0.30$, with tank. Limits for different $\omega/\omega_{\eta}$ for $m_t=3\%m$ and $\eta_t=0.2$.

Figure 9 – Ship TS, $GM=0.37$, $Fn=0.30$, with tank. Limits for different $\omega/\omega_{\eta}$ for $m_t=3\%m$ and $\eta_t=0.3$. 
Figure 10 – Ship TS, $GM=0.37$, $Fn=0.30$, with tank. Limits for different $m_i$ for $\omega_i = \omega_{n4}$ and $\eta_i = 0.2$.

Figure 11 – Ship TS, $GM=0.37$, $Fn=0.30$, with tank. Limits for different $m_i$ for $\omega_i = \omega_{n4}$ and $\eta_i = 0.3$.